CITIC-UGR

Universidad de Jaén

SEFORI 2016

VI SEMINARIOS DE FORMACIÓN PARA LA INVESTIGACIÓN EN TIC

Computación con Palabras aplicada a la Toma de Decisiones

Luis Martínez López

martin@ujaen.es

http://sinbad2.ujaen.es

SINBAD²

Intelligent Systems Based on Fuzzy Decision Analysis

UJA.es

OUTLINE

- **INTRODUCTION**
- **COMPUTING WITH WORDS**
- **COMPUTING WITH WORDS IN DECISION MAKING**
- **OPEN RESEARCH**
- **CONCLUSIONS**

INTRODUCTION

Decision Making: Introduction

- **What is Decision Making ?**

Decision Making has been the subject of active research in many different fields and studied from several perspectives.

Daniel Kahneman, Amos Tversky (2000). *Choice, Values, Frames*. The Cambridge University Press

Decision Making (DM) is a complex process and a key activity proper of human beings. Some authors claim that DM in complex situations is a clear distinction between human beings and animals

Bouyssou D. et al., Evaluation and Decision Models. A critical perspective. Kluwer's International series. 2000

DM is the act or process of deciding something especially with a group of people

Merriam-Webster

Decision Making: Introduction

- **Decision Making**
  - In short:
    - Decision making is an activity that searches a solution fulfilling the constraints of a problem among several alternatives
    - Decision making implies **reasoning processes** that could be either **rational** or **emotional.** Hence:
      - Decision making may be **rational or emotional**
    - Decision making is a complex process proper of human beings.

Decision Making: Introduction

Basic Elements of a Classical Decision Problem

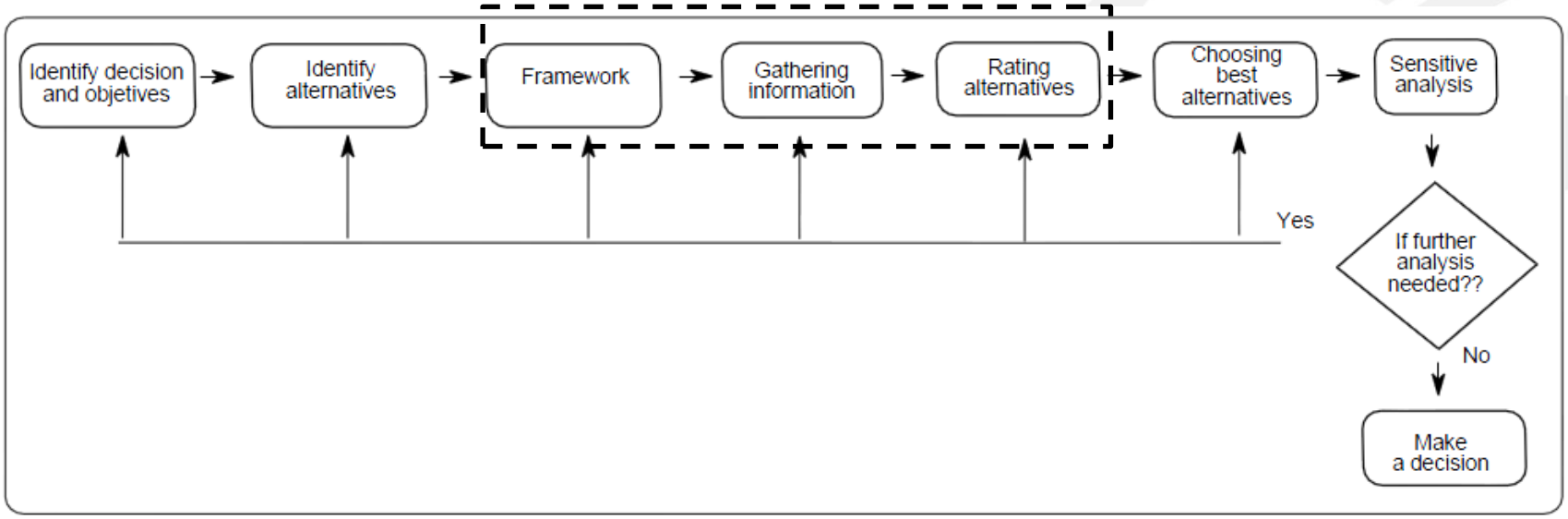
- A set of alternatives or available decisions: $A=\{a_1,\ldots,a_m\}$
- A set of *states of nature* that defines the framework of the problem: $S=\{s_1,\ldots,s_n\}$
- A set of utility values, u_{ij}, each one associated to a pair composed of an alternative and a state of nature: $u_{ij}:\langle a_i,s_j\rangle$
- A function that establishes the expert's preferences regarding the plausible results.

	s_1	...		s_N
Alternative 1	u_{11}	u_{12}	...	u_{1N}
Alternative 2	u_{21}	u_{22}	...	u_{2N}
...	...	...	...	...
Alternative M	u_{M1}	u_{M2}	...	u_{MN}

Decision Making: Introduction

Classical Decision Solving Process

- Scientific literature:
  - Decision analysis techniques have facilitated the resolution of complex decision making problems

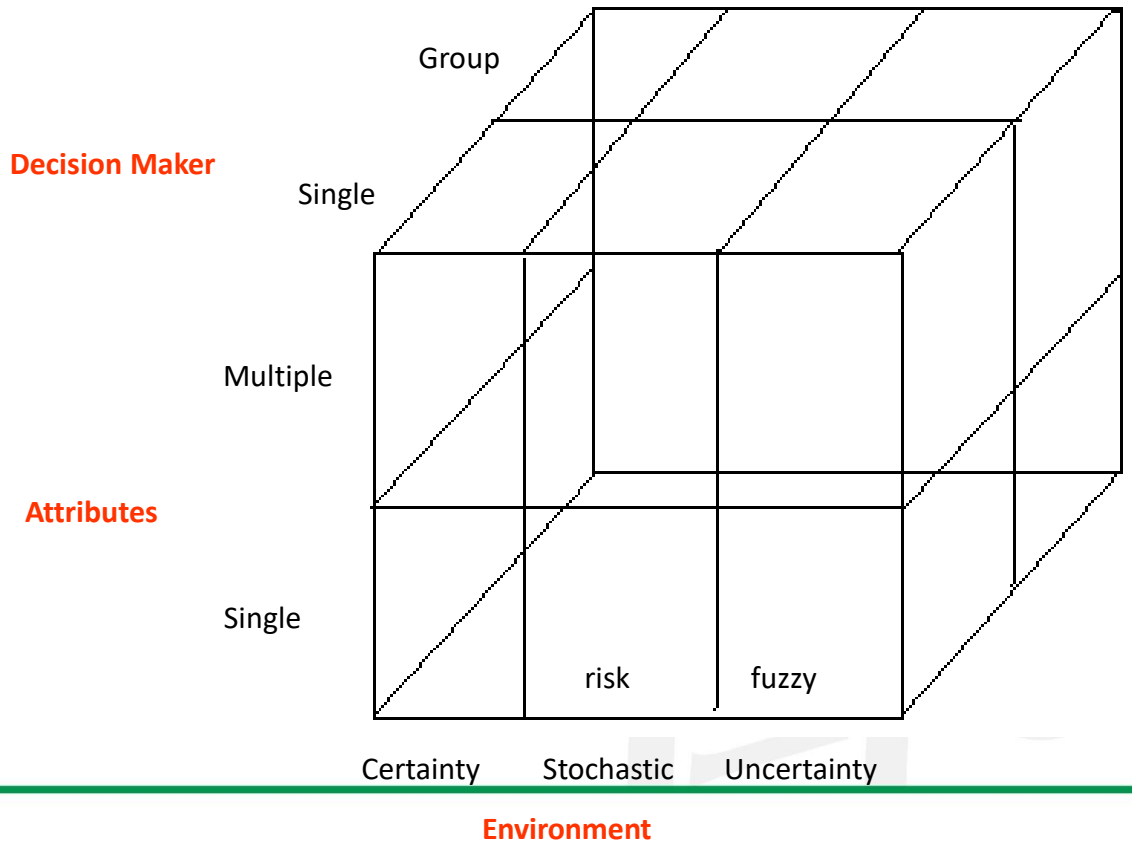


Decision Making: Classification

- Characterization
  - Dimensions
    - Number of ***attributes/criteria***
      - Single
      - Multiple
    - Number of ***decision makers***
      - Single
      - Multiple
    - ***Environment***
      - Certainty
      - Stochastic / Risk
      - Uncertainty

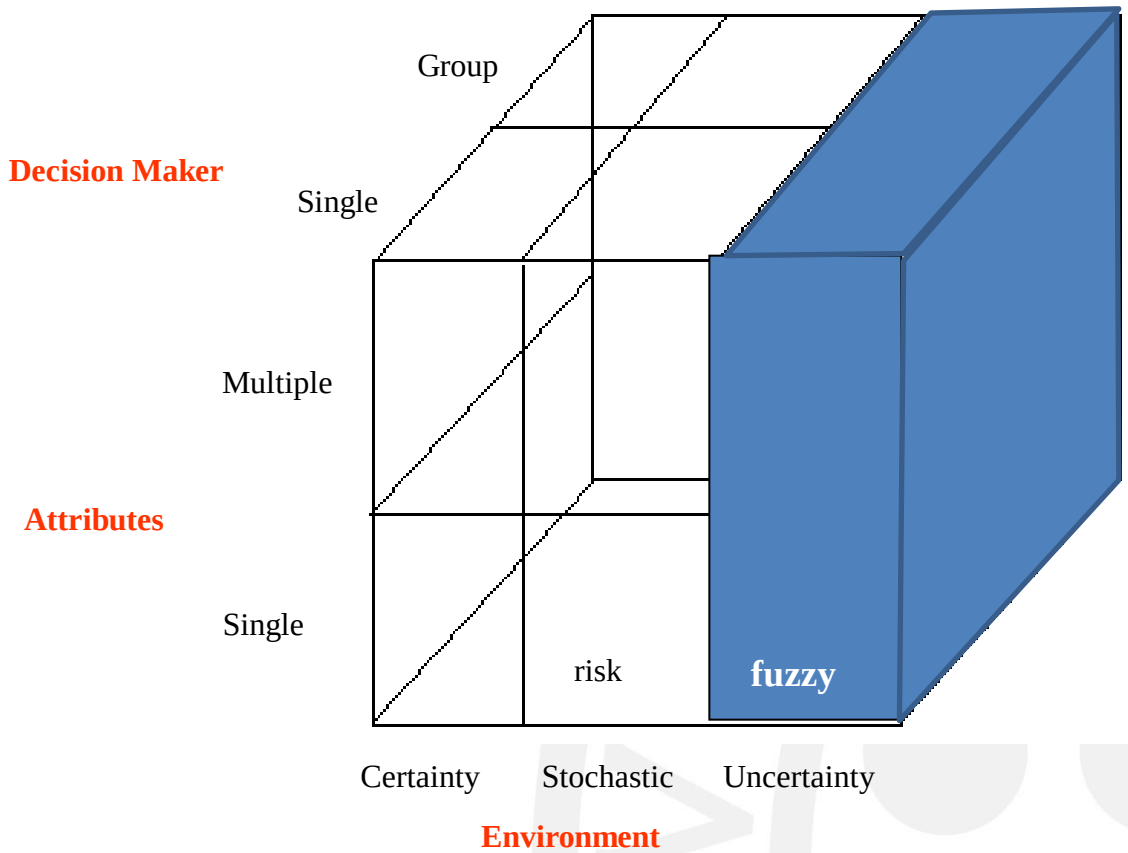
Decision Making: Classification

- Dimensions



Decision Making: Classification

- **Dimensions**



Decision Making: Introduction

- **DM under Uncertainty**
  - Various types are commonly met in real world DM problems
    - Considering uncertainty when building mathematical models
      - Increase credibility and efficiency of decisions
  - Starting point
    - Strict correspondence between models and level of uncertainty
      - Adequacy of modelling to obtain real impact of solving problem
      - Any simplification of reality distorts the nature of the problem
      - Doubts:
        - » Validity of using uncertainty factors within framework of traditional approaches
          - Deterministic
          - Probabilistic
        - » Not ensure sufficient adequate consideration of uncertainty factor in the DM

Decision Making: Introduction

- **DM under Uncertainty**
  - New approaches based on different mathematical models
    - Outstanding: Fuzzy sets theory
      - Open a new avenue of giving up "excessive" precision
        - » Traditional modelling approaches
        - » Preserving reasonable rigor
      - ***Principle of incompatibility***: trade-offs between precision and relevance of models

"…as the complexity of the system increases, our ability to make precise and yet significant statement on its behavior diminishes until a threshold beyond which precision and relevance become almost mutually exclusive characteristics".

Zadeh, L. A. (1973), 'Outline of a new approach to the analysis of complex systems and decision processes', IEEE Trans. SMC 1, 28–44.

      - Operating in fuzzy space
        - » Intuitive aspects of qualitative analysis
        - » Obtaining reliable quantitative information

Decision Making: Introduction

- **DM under Uncertainty**
  - Fuzzy Sets Theory
    - Reflects adequately the essence of decision making process
      - Human Factor in real world DM
        - » Important effect
        - » Visible position
      - Considering human thinking
        - » Perceptions, preferences:
          - Vague
          - Subjective
        - » Linguistic information

Decision Making: Introduction

- **DM under Uncertainty**
  - Fuzzy sets support quantifying ***linguistic facet*** *in preference modelin*
    - *Applied to Decision making*
  - Application of Fuzzy Sets to preference modeling and analysis of DM
    - Flexible environment to deal with inherent fuzziness of perception
    - Incorporation more human consistency into preference models
    - Improving adequacy, enhancing credibility and efficiency
    - Highly beneficial:
      - For formation of convincing and affective human oriented interfaces
        - » DSS and human beings

COMPUTING WITH WORDS

COMPUTING WITH WORDS

- *Human beings*
  - Many remarkable capabilities
  - Two stand out
    1. Capability to *converse, communicate, reason and make decisions* in an environment of imprecision, uncertainty and partiality of truth
    2. Capability to perform a wide variety of *physical and mental tasks without any measurements*

COMPUTING WITH WORDS

Human Beings in their daily life activities

- Employ mostly words in computing and reasoning
  - Arriving at conclusions expresses in words from linguistic premises

COMPUTING WITH WORDS

Mendel J., **Zadeh L.A.**, Trillas, E., Lawry J., Hagras H., Guadarrama, S. "What computing with words means to me". Computational Intelligence Magazine. IEEE, **2010**. V. 5, n. 1, pp. 20-26

To achieve human level machine intelligence it is necessary to emulate such capabilities

COMPUTING WITH WORDS

Methodology in which words are used in place of numbers for computing and reasoning

Lotfi A. Zadeh. Fuzzy Logic = Computing with Words. *IEEE Transactions on Fuzzy Systems, Vol. 4, No. 2, May **1996***.

COMPUTING WITH WORDS

- CW means different things for different people
  - *Methodology in which the objects of computation are words and propositions drawn from a natural language for reasoning, computing and decision making with linguistic information*
  - *Methodology to include human sourced information in the formal computer based decision-making models*
  - The incorporation of vague linguistic concepts into intelligent computer systems

Mendel J., **Zadeh L.A.**, Trillas, E., Lawry J., Hagras H., Guadarrama, S. "What computing with words means to me". Computational Intelligence Magazine. IEEE, **2010**. V. 5, n. 1, pp. 20-26

COMPUTING WITH WORDS

- *L.A. Zadeh* underlying CW are three main rationales:
  - Much of human knowledge is linguistically described
  - Words a less precise than numbers, therefore CW could be powerful tool to deal with imprecise information
  - Precision carries a cost. If there is a tolerance for imprecision it can be exploited by using words in place of numbers

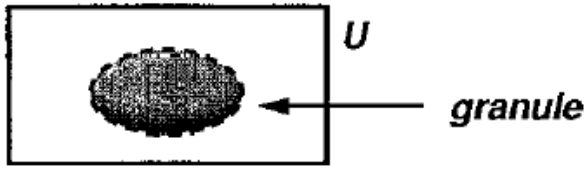
COMPUTING WITH WORDS

- Foundations of CW were rooted much earlier:

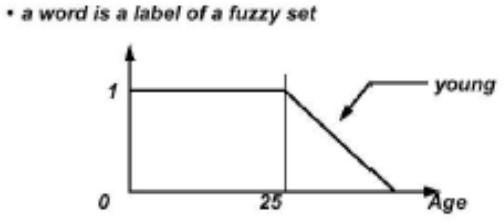
L. Zadeh, “Outline of new approach to the analysis of complex systems and decision processes” IEEE SMC, vol SMC-3, **1973**, 28-44.

Lotfi A. Zadeh. The concept of a linguistic variable and its application to approximate reasoning, Part I: Inf. Sci. 8, 199-249, 1975; Part II: Inf. Sci. 8, 301-357, 1975; Part III: Inf. Sci. 9, 43-80, **1975**.

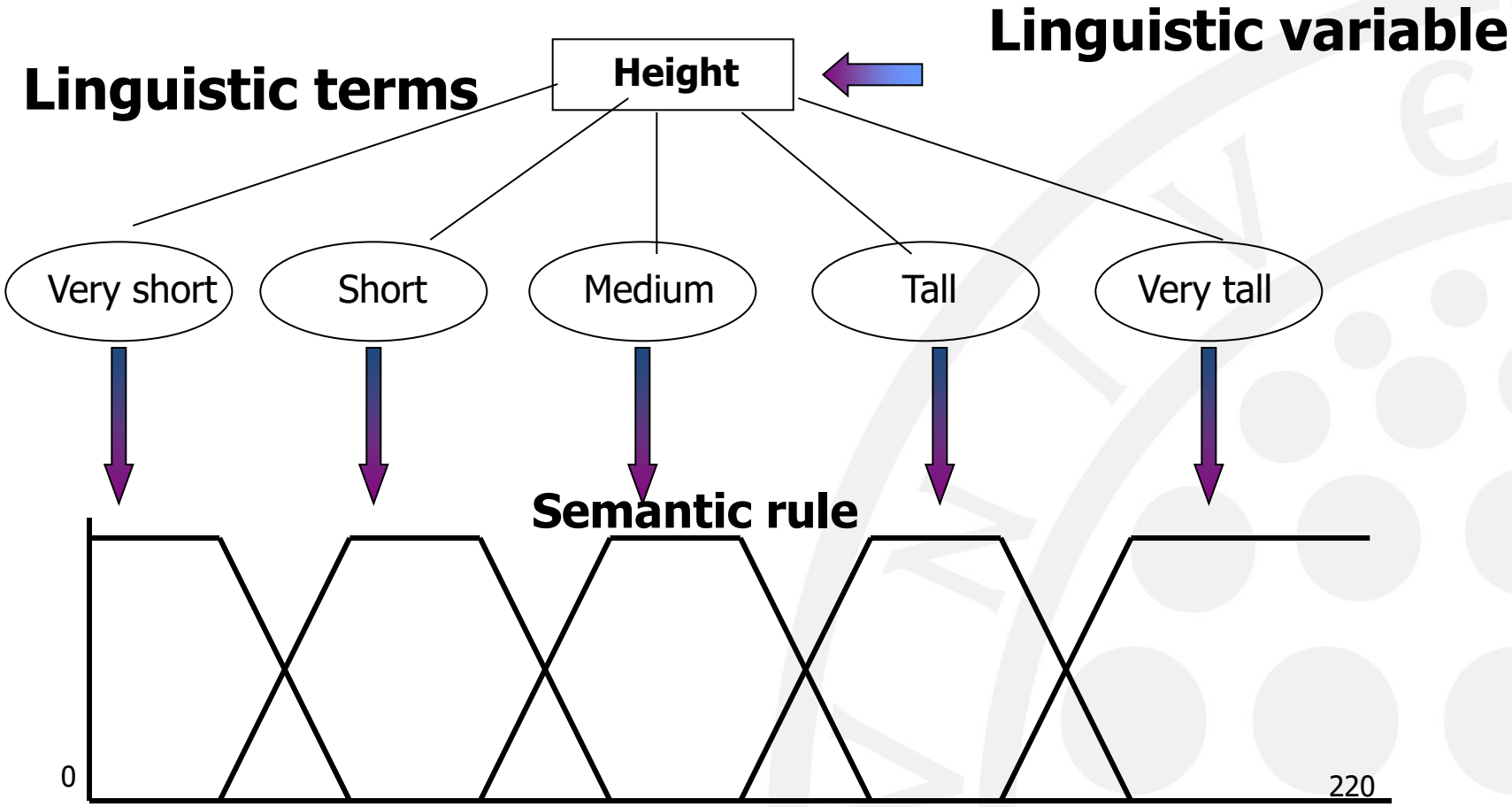
a granule is a clump of objects (points) drawn together by indistinguishability, similarity, proximity or functionality



Linguistic Variable
<L, T(L), U, S, M>



COMPUTING WITH WORDS



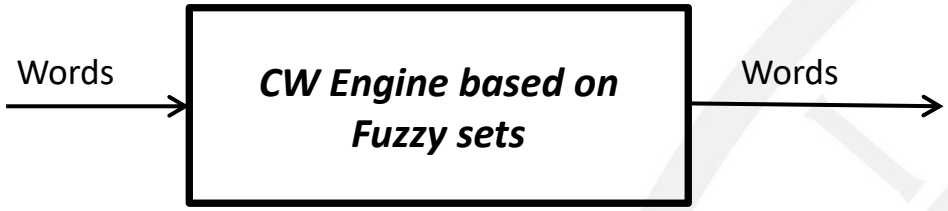
COMPUTING WITH WORDS

- Fuzzy logic plays a pivotal role in CW
  - Foundations previous to CW
  - Fuzzy logic provides the machinery to achieve the objectives of CW

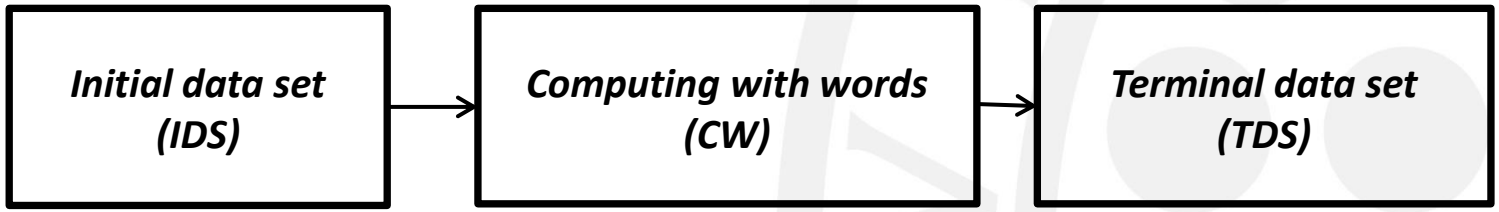
COMPUTING WITH WORDS

- Computational theory

Arriving at conclusions expresses in words from linguistic premises



  – There is a great deal of computing with numbers in CW
    - Unseen by the user



COMPUTING WITH WORDS

Computing with Words is a methodology for reasoning, computing and decision-making with information described in natural language.

COMPUTING WITH WORDS
IN
DECISION MAKING

CW IN DECISION MAKING

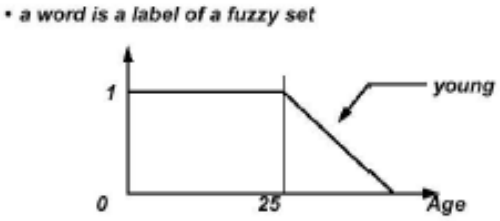
- *Why Computing with Words in Decision Making ?*
  - CW narrow differences human reasoning and computing

Manipulation of imprecise, vagueness or partial issues is usual in human reasoning and Decision Making

  - CW enrich decision models
    - Modelling uncertainty with linguistic information
      - Related to imprecision and vagueness of meanings
    - Enhances the reliability and flexibility decision models

CW IN DECISION MAKING

Linguistic Variable
<L, T(L), U, S, M>

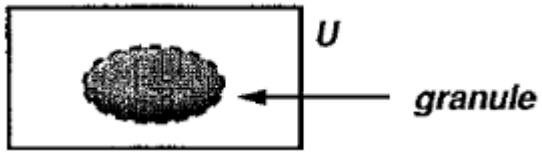


- Semantics of fuzzy sets could have three interpretations:

SIMILARITY

$\mu_F(u)$ is the degree of proximity of u to prototype elements of F

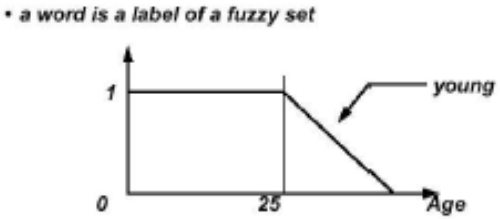
Interest in classification



D. Dubois and H. Prade. The three semantics of fuzzy sets. Fuzzy Sets and Systems 90 (1997) 141-150.

CW IN DECISION MAKING

Linguistic Variable
<L, T(L), U, S, M>



- Semantics of fuzzy sets could have three interpretations:

UNCERTAINTY

$\mu_R(u)$ is the degree of possibility that a parameter x has value u, given that all that is known about it is that "x is R".

Interest in expert systems and AI

X is R

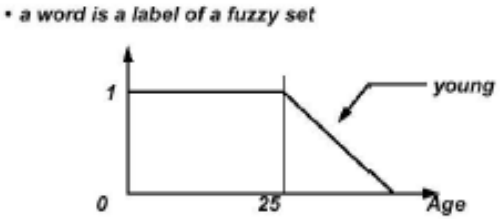
$\Pi_X = R$

$\mathrm{Poss}\{X = u\} = \mu_R(u)$

D. Dubois and H. Prade. The three semantics of fuzzy sets. Fuzzy Sets and Systems 90 (1997) 141-150.

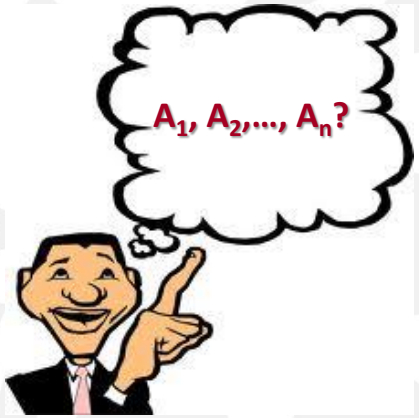
CW IN DECISION MAKING

Linguistic Variable
<L, T(L), U, S, M>



- Semantics of fuzzy sets could have three interpretations:

It seems natural and usual the use of linguistic labels to express the intensity of preference for a given alternative



D. Dubois and H. Prade. The three semantics of fuzzy sets. Fuzzy Sets and Systems 90 (1997) 141-150.

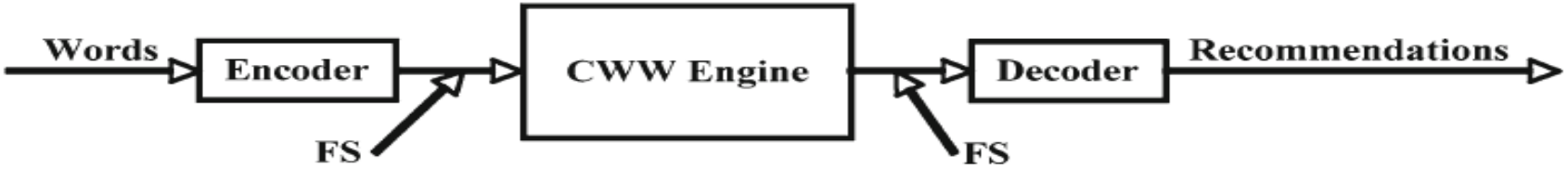
CW IN DECISION MAKING

- *Early CW in Decision Making*

  R.M. Tong and P.P. Bonissone. A linguistic approach to decision making with fuzzy sets. *IEEE Transactions on Systems, Man and Cybernetics*, SMC-10(11):716–723, 1980

  – Linguistic assessments make explicit subjective nature of any choice

  – The preference of one alternative over the others can be expressed as a truth qualified proposition



CW IN DECISION MAKING

- *Early CW in Decision Making*

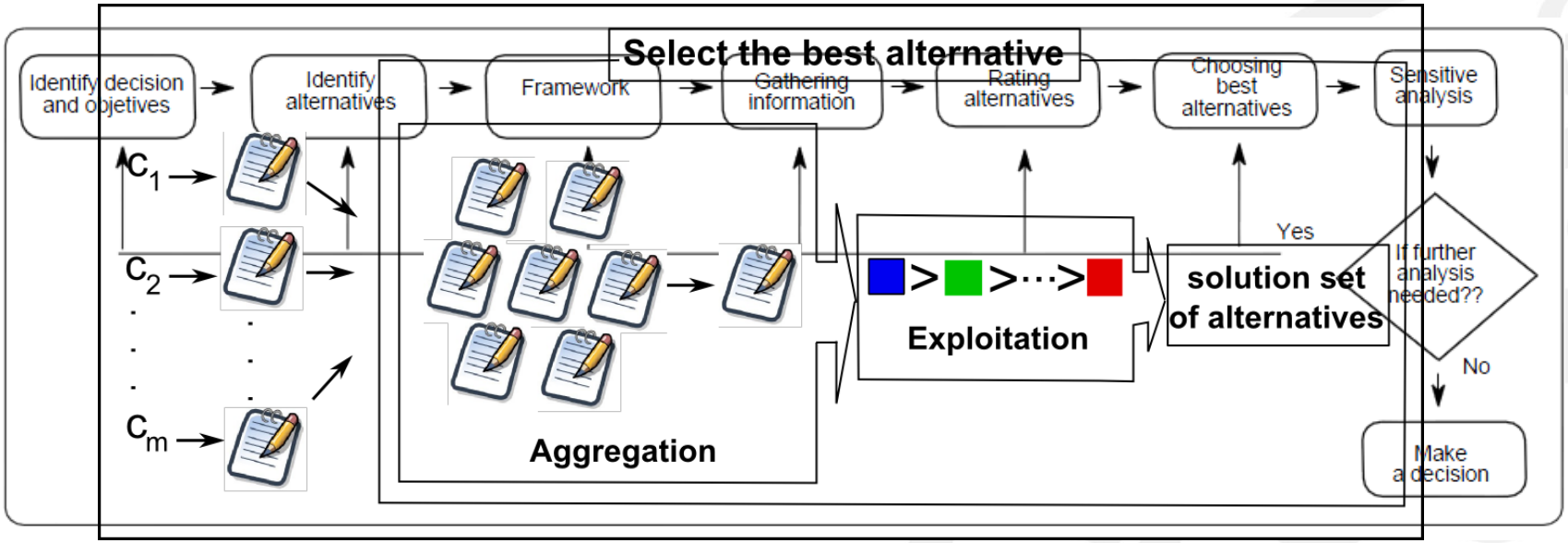
  R.R. Yager. A new methodology for ordinal multiple aspect decisions based on fuzzy sets Decision Sciences 12 (1981) 589-600
  R.R. Yager. Non-numeric multi-criteria multi-person decision making. Group Decision and Negotiation 2 (1993) 81-93

  – The values for evaluating ratings and importance expressed in a linguistic scale.
  – Easier for the evaluator to provide the evaluation
  – Ordinal scales: *Symbolic approach, max-min*

J.J. Buckley. The multiple judge, multiple criteria ranking problem: A fuzzy set approach. Fuzzy Sets and Systems, 13(1):25–37, 1984

CW IN DECISION MAKING

- *Decision Solving Scheme*



  - Aggregation
  - Exploitation

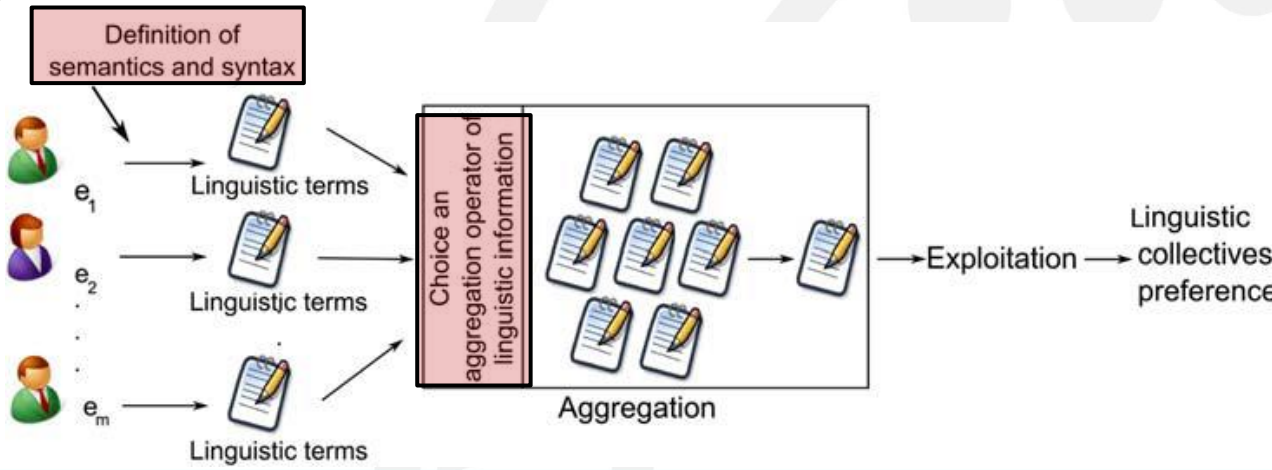
Roubens, M. (1997), 'Fuzzy sets and decision analysis,' Fuzzy Sets and Systems, 90, 199–206.

CW IN DECISION MAKING

- *Linguistic Decision Solving Scheme*

  F. Herrera, E. Herrera-Viedma, Linguistic decision analysis: Steps for solving decision problems under linguistic information, Fuzzy Sets and Systems 115 (1) (2000) 67–82

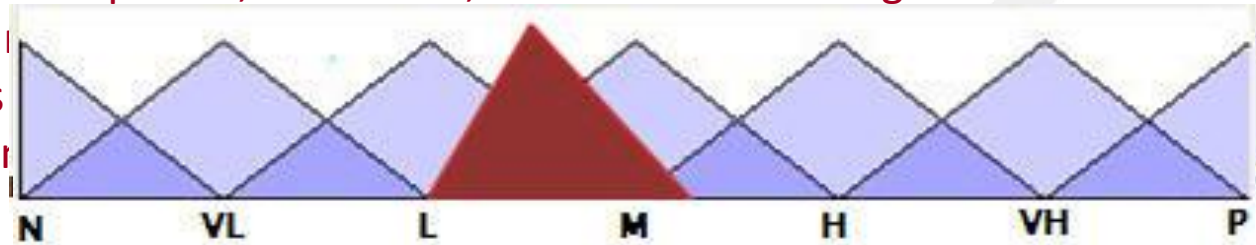
  – *Choice Linguistic term sets and semantics*
  – *Choice linguistic aggregation operator*
  – *Resolution Scheme*



CW IN DECISION MAKING

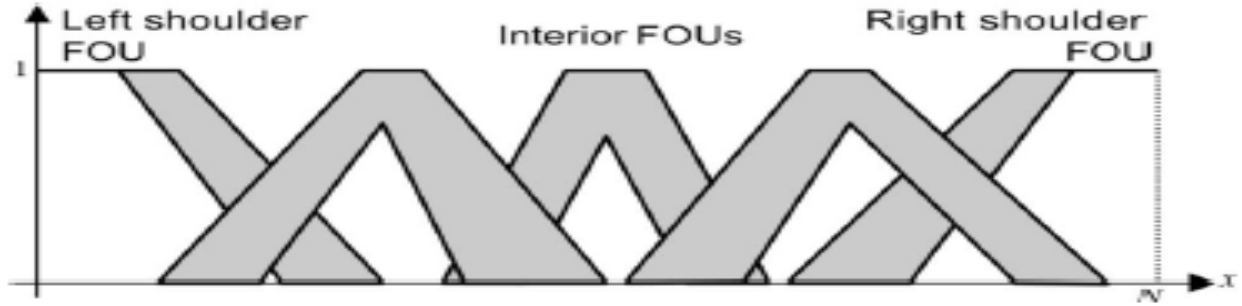
- ***Linguistic Computational Models***
  - Semantics models
    - Based on membership functions (type 1 fuzzy sets)

- K. Anagnostopoulos, H. Doukas, and J. Psarras. A linguistic multicriteria analysis system combini selection. Expert Systems
- R. Degar clinical decision making.



    - Based on type-2 fuzzy sets

- W. Dong verage and interval 5–1161, 2007
- S.M. Zhc rtain informat onal Journal of Intelligen



CW IN DECISION MAKING

- ***Linguistic Computational Models***
  - Symbolic Models
    - Based on ordinal scales
      - Based on ordinal scales and max-min operators
      - Based on convex combinations
      - Based on virtual linguistic terms

- R.R. Yager. An approach to ordinal decision making.International Journal of Approximate Reasoning, 12:237–261, 1995.
- M. Delgado, J.L. Verdegay, and M.A. Vila. On aggregation operations of linguistic labels. International Journal of Intelligent Systems, 8(3):351–370, 1993.
- Z.S. Xu. A method based on linguistic aggregation operators for group decision making with linguistic preference relations. Information Sciences, 166(1-4):19–30, 2004.

CW IN DECISION MAKING

- ***Linguistic Computational Models***
  - Symbolic Models extending the use of indexes:
    - Linguistic 2-tuple model
    - Proportional 2-tuple : An extension of the 2-tuple model
    - Others 2-tuple based linguistic computational models

- F. Herrera and L. Martínez. A 2-tuple fuzzy linguistic representation model for computing with words. IEEE Transactions on Fuzzy Systems, 8(6):746–752, 2000
- J.H. Wang and J. Hao. A new version of 2-tuple fuzzy linguistic representation model for computing with words. IEEE Transactions on Fuzzy Systems, 14(3):435–445, 2006.
- Y. Dong, Y. Xu, and S. Yu. Computing the numerical scale of the linguistic term set for the 2-tuple fuzzy linguistic representation model. IEEE Transactions on Fuzzy Systems, 17(6):1366–1378, 2009.

CW IN DECISION MAKING

- ***Linguistic Computational Models***
  - Symbolic Models
    - Computing with Words
      - Some models should not be considered within CWW
    - Comparative survey

- R.M. Rodríguez, L. Martínez, An Analysis of Symbolic Linguistic Computing Models in Decision Making. International Journal of General Systems, vol. 42, issue 1, pp. 121-136, 2013.

CW IN DECISION MAKING

- ***Applications***
  - ***Industrial Applications***

Application	Authors	Year
Supplier selection and evaluation	Awasthi A. and Kannan G.	2016
	Lin, Qing-Lian et al.	2013
Location Selection	Demirel et al.	2010
	Lin, J. et al.	2015
Material, stock and system selection	Tsai, Chih-Fong; Chen, Zong-Yao	2013
	Peng, A. et al.	2015

CW IN DECISION MAKING

- ***Applications***
  - ***Resource Management***

Application	Authors	Year
Sustainable development	Colin, Francois et al.	2011
	Doukas H.	2013
	Hu Z., et al.	2015
Human resources	Meng D. and Pei Z.	2011
	Afshari, Ali Rez et al.	2013
	Manoharan, T. R et al.	2013

CW IN DECISION MAKING

- ***Applications***
  - ***Internet Based***

Application	Authors	Year
Recommender Systems	Noguera J.M. et al.	2013
	Yera R. et al.	2016
Web Quality	Morente-Molinera, J. A. et al.	2016
	Cebi, Selcuk	2013
Social Networks	Yager, R. R.; Reformat, M. Z.	2013
	Cid-Lopez, A. et al.	2015

OPEN RESEARCH FOR

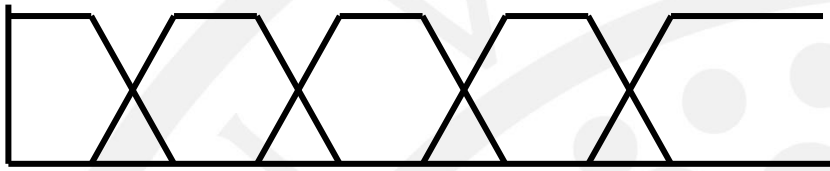
COMPUTING WITH WORDS IN DECISION MAKING

CHALLENGES

- CW in Decision Making
  - Example

The consulting company has a group of four consultancy departments

p_1	p_2	p_3	p_4
Cost analysis	System analysis	Risk analysis	Techonology analysis.



		alternatives			
	L_{ij}	x_1	x_2	x_3	x_4
experts	p_1	VL	M	M	L
	p_2	M	L	VL	H
	p_3	H	VL	M	M
	p_4	H	H	L	L

x_1	x_2	x_3	x_4	
M	M	L	M	$\{x_1, x_2, x_4$
M	M	L	M	$\{x_1, x_2, x_4$

CHALLENGES

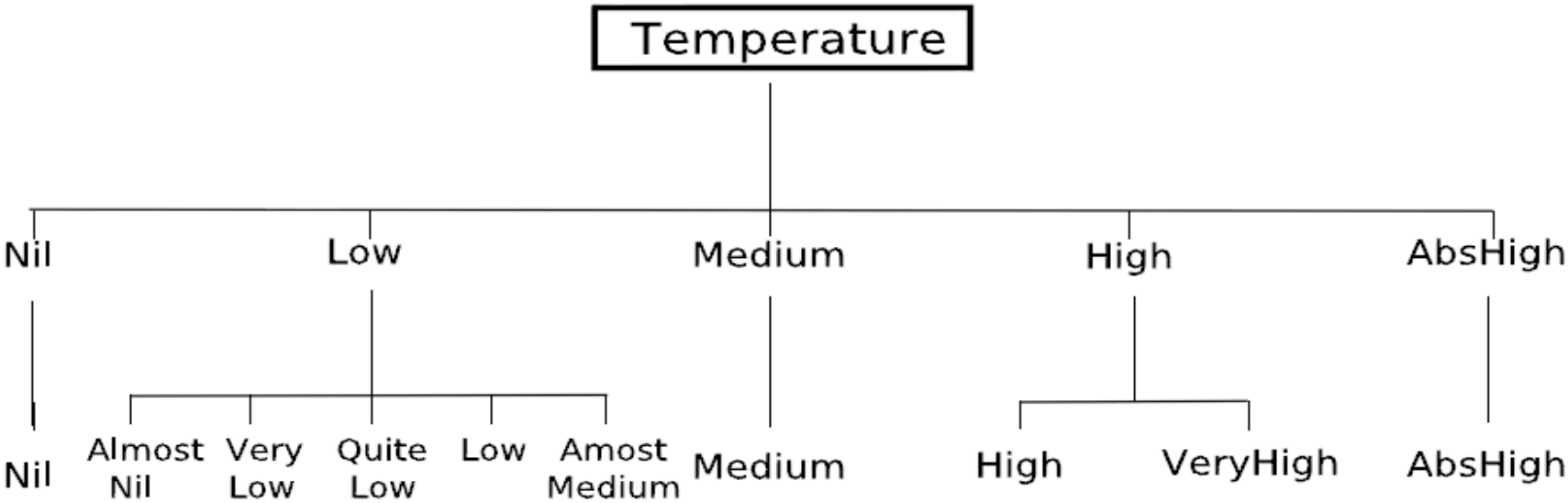
- CW in Decision Making
  - **Linguistic terms meaning**
    - Means different things for different people
    - Modelling such a matter of fact
  - **Enrich vocabulary**
    - Atomic linguistic terms
      - Not enough
    - Composite terms
    - How to manage and build composite terms

CHALLENGES: Meaning

- Words can mean different things for different people
  - Use of fuzzy sets to model a word
  - Capture word uncertainties
- Reasons
  - Different degree of knowledge
  - Uncertainty

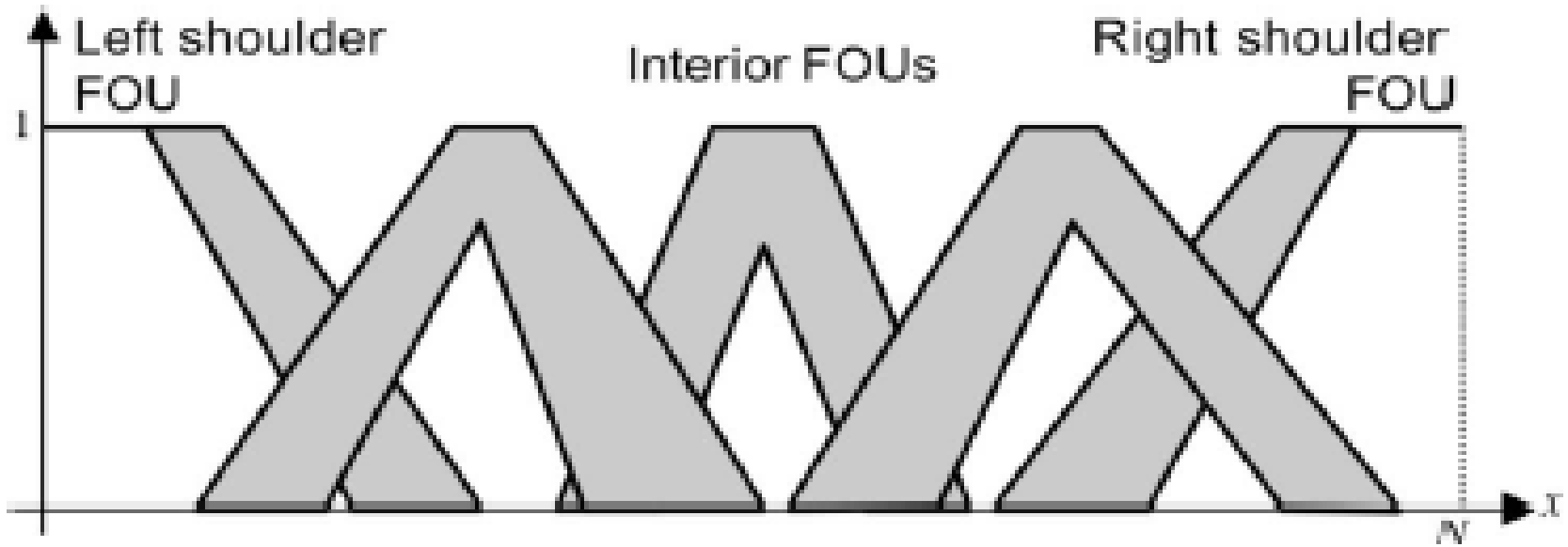
CHALLENGES: Meaning

- ***Degree of Knowlegde***
  - Provide multiple linguistic term sets
    - According to the degree of knowledge



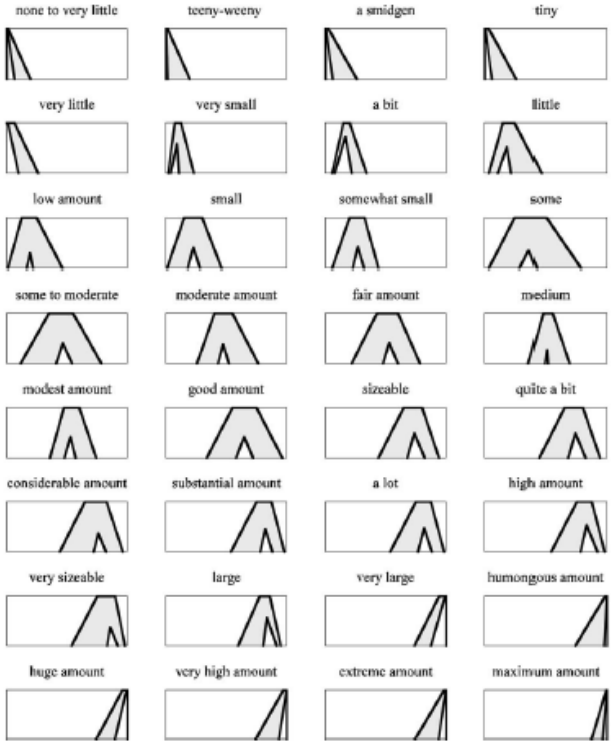
CHALLENGES: Meaning

- ***Uncertainty***
  - Provide type 2 fuzzy set representation

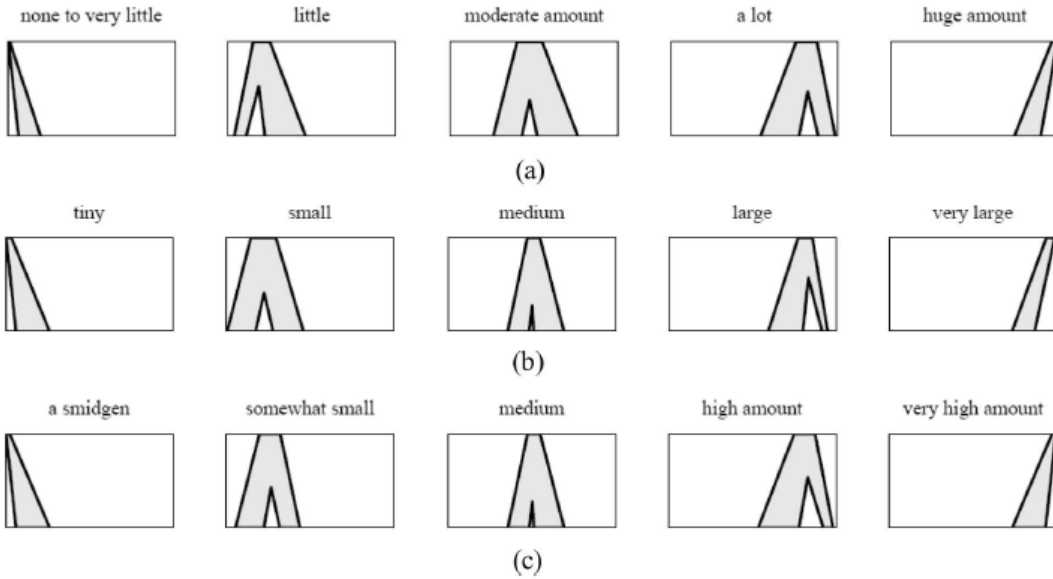


CHALLENGES: Meaning

- Uncertainty
  - Codebook



Sub-Vocabulary



CHALLENGES: Meaning

- **Still different meanings under one representation**
- Recent Proposal: *"Personalized Individual Semantics"*
  - Previous models
    - Useful in many problems
    - Do not represent the specific semantics of individuals
  - For representing individual semantics

Li, C-C., Y.C. Dong, F. Herrera, E. Herrera-Viedma, and L. Martínez, "Personalized individual semantics in Computing with Words for supporting linguistic Group Decision Making. An Application on Consensus reaching", Information Fusion, vol. 33, issue 1, pp. 29-40, 2017.

- Based on the interval numerical scale
- CW dealing with PIS based on the 2-tuple linguistic model

CHALLENGES: *Enrich vocabulary*

- Enrich vocabulary
  - Atomic
  - Composite
- Linguistic terms defined a priori
  - Atomic
    - Single semantics
  - Composite
    - Single semantics
    - Modification of atomic semantics
- Limitation to provide preferences

CHALLENGES: Enrich vocabulary

- To avoid this limitation
  - Different proposals facilitate the elicitation of preferences
    - Complex linguistic expressions

J.H. Wang and J. Hao. A new version of 2-tuple fuzzy linguistic representation model for computing with words. *IEEE Transactions on Fuzzy Systems*, 14(3):435–445, 2006.

Y. Tang and J. Zheng. Linguistic modelling based on semantic similarity relation among linguistic labels. *Fuzzy Sets and Systems*, 157(12):1662–1673, 2006.

J. Ma, D. Ruan, Y. Xu, and G. Zhang. A fuzzy-set approach to treat determinacy and consistency of linguistic terms in multi-criteria decision making. *International Journal of Approximate Reasoning*, 44(2):165–181, 2007.

G. Zhang, Y. Dong, and Y. Xu. Consistency and consensus measures for linguistic preference relations based on distribution assessments. *Information Fusion*, 17(0):46–55, 2014.

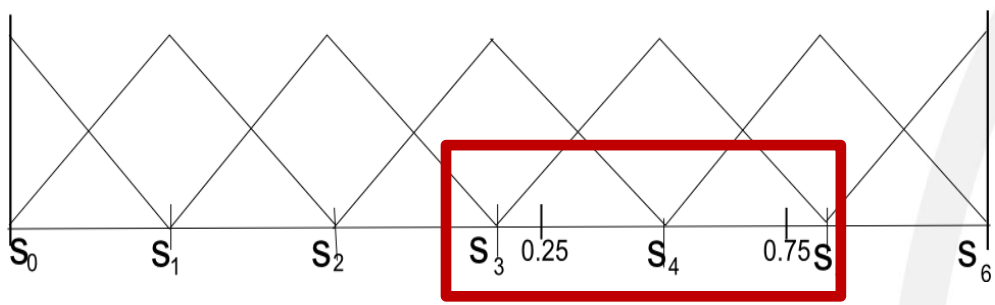
Rodríguez, R M., Á. Labella, and L. Martínez, "An Overview on Fuzzy Modelling of Complex Linguistic Preferences in Decision Making", International Journal of Computational Intelligence Systems , vol. 9: Taylor and Francis, pp. 81-94, 2016.

CHALLENGES: Enrich vocabulary

1. Proportional 2-tuple linguistic model (Wang and Hao, 2006)

- Preferences
  - Proportions of two linguistic terms
- Symbolic proportion

$$(\alpha, s_i), (\beta, s_{i+1}) \in S$$



{(high,0.25),(very high, 0.75)}
{(low, 0.37),(medium,0.63)}

$$(0.25, s_3), (0.75, s_4)$$

CHALLENGES: Enrich vocabulary

2. Linguistic model based on semantic similarity relation among linguistic labels (Tang and Zheng 2006)

- Allows to define *Linguistic expressions*

The set of linguistic expressions *LE*, is defined recursively

1) $s_i \epsilon LE$ for k= $\{1, \dots g\}$

2) If $\theta, \phi \in LE$ then $\neg\theta, \theta \vee \phi, \theta \wedge \phi, \theta \rightarrow \phi \in LE$

- Example

S={s_0:nothing,s_1:very_low,s_2:low,s_3:medium,s_4:high,s_5:very_high,s_6:perfect}

$$\mathbf{LE_1} = \neg \boldsymbol{high} \vee \boldsymbol{medium}$$

$$\mathbf{LE_2} = \boldsymbol{medium} \wedge \boldsymbol{high}$$

CHALLENGES: Enrich vocabulary

3. A fuzzy-set approach to treat determinacy and consistency of linguistic terms (Ma et al 2007)

   – Increase the flexibility
   – Multiple linguistic terms → synthesized comments

Neither	Very low	Low	Medium	high	Very high	Absolute	Comment
0	0	1	1	0	0	0	*Commonly*
0	0	0	0	0	1	1	*Excellent*

   – **No rule to fix the syntax**
   – Computations
     - A fuzzy model
     - Determinacy and consistency measures

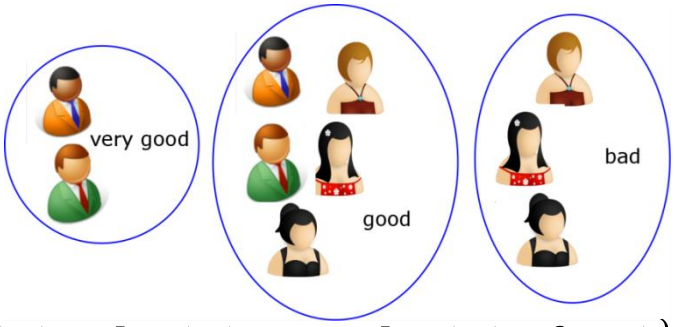
CHALLENGES: Enrich vocabulary

4. Linguistic distribution (Zhang et al 2014)
   - Generalization of proportional 2-tuple linguistic model
   - Including distribution assessment
     - Assign symbolic proportions to all the terms
     - A distribution assessment

$$m=\{(s_i,\beta_i)\,|\,i=\{0,\ldots,g\}\}$$

where S_i $s_i \in S=\{s_0,\ldots,s_g\}, \beta_i \geq 0, \sum_{i=0}^{g}\beta_i=1$, and β_i is the symbolic proportion of

- Example: panel evaluation
  - 10 panelists (distribution assessment)



$$\{(\text{nothing},0),(\text{very bad},0),(\text{bad},0.3),(\text{medium},0),(\text{good},0.5),(\text{very good},0.2),(\text{perfect},0)\}$$

CHALLENGES: Enrich vocabulary

J.H. Wang and J. Hao. A new version of 2-tuple fuzzy linguistic representation model for computing with words. *IEEE Transactions on Fuzzy Systems*, 14(3):435–445, 2006.

Y. Tang and J. Zheng. Linguistic modelling based on semantic similarity relation among linguistic labels. *Fuzzy Sets and Systems*, 157(12):1662–1673, 2006.

J. Ma, D. Ruan, Y. Xu, and G. Zhang. A fuzzy-set approach to treat determinacy and consistency of linguistic terms in multi-criteria decision making. *International Journal of Approximate Reasoning*, 44(2):165–181, 2007.

G. Zhang, Y. Dong, and Y. Xu. Consistency and consensus measures for linguistic preference relations based on distribution assessments. *Information Fusion*, 17(0):46–55, 2014.

- These proposals
  - **Linguistic expressions far away** from human beings
  - **No formalization**

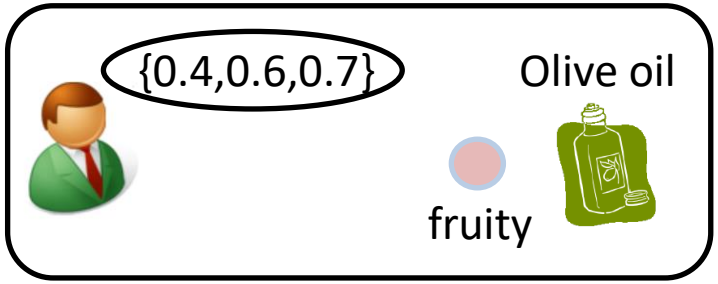
CHALLENGES: Enrich vocabulary

- Looking for linguistic expressions
  - Close to human beings
  - Flexible and not a priori
    - Composite terms
  - Multiple terms
    - Hesitant situations

R.M. Rodríguez, L. Martínez, F. Herrera, Hesitant Fuzzy Linguistic Term Sets for Decision Making. IEEE Transactions on Fuzzy Systems, vol. 20, issue 1, pp. 109-119, 2012.

CHALLENGES: Enrich vocabulary

- Hesitant fuzzy sets, HFS
  - Hesitant situations
  - Quantitative contexts
  - Experts hesitate among different values
    - Fix membership degree
- Example sensory evaluation



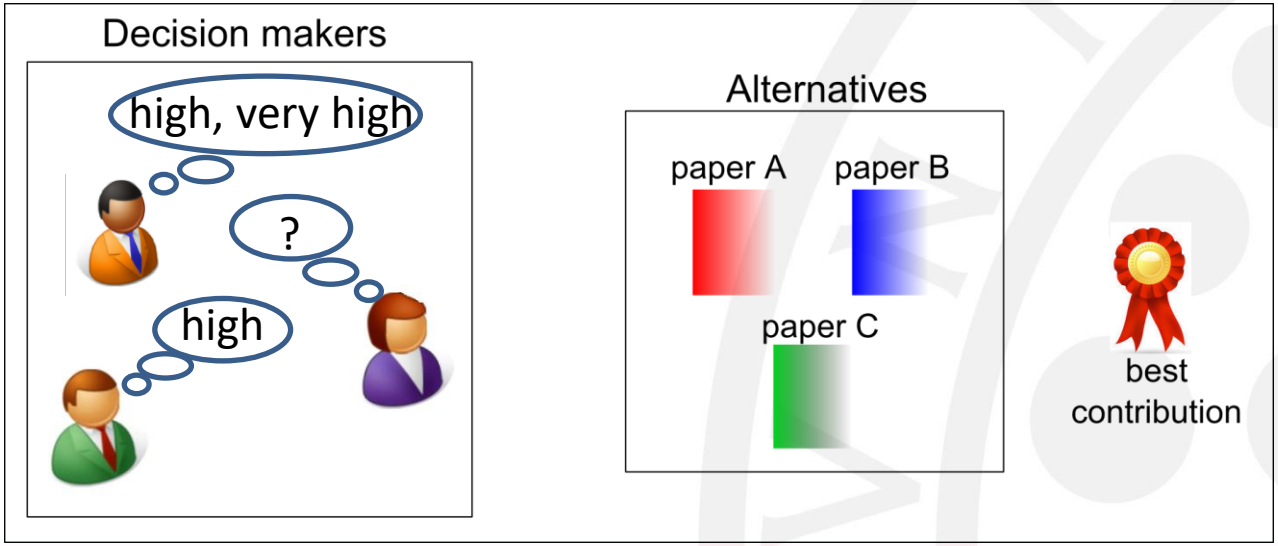
V. Torra, Hesitant fuzzy sets, International Journal of Intelligent Systems 25 (6) (2010) 529–539.

Rodríguez, R M., B. Bedregal, H. Bustince, Y.C. Dong, B. Farhadinia, C. Kahraman, L. Martínez, V. Torra, Y.J. Xu, Z.S. Xu, F. Herrera"A Position and Perspective Analysis of Hesitant Fuzzy Sets on Information Fusion in Decision Making. Towards High Quality Progress", Information Fusion, vol. 29, pp. 89-97, 2016.

CHALLENGES: Enrich vocabulary

- Hesitant situations
  - Linguistic contexts
  - Different linguistic values
- Example

  Award the best paper in an International Conference



CHALLENGES: Enrich vocabulary

- **Hesitant fuzzy linguistic term sets (HFLTS)**
  - Multiple linguistic terms
  - Fuzzy linguistic approach
  - Hesitant fuzzy sets
- Definition

Let $S=\{s_{0,\ldots,}s_g\}$ be a linguistic term set, a HFLTS, Hs, is an ordered finite subset of consecutive linguistic terms of S

$H_S = \{s_i, s_{i+1}, \ldots, s_j\}$, such that $s_k \in S, \quad k \in \{i, \ldots, j\}$

CHALLENGES: Enrich vocabulary

- **Hesitant fuzzy linguistic term sets (HFLTS)**
  - Multiple linguistic terms
  - Fuzzy linguistic approach
  - Hesitant fuzzy sets
- Definition
- Example

S={s_0:nothing,s_1:very low,s_2:low,s_3:medium,s_4:high,s_5:very high,s_6:perfect}

H_S={very low, low, medium}

CHALLENGES: Enrich vocabulary

- HFLTS → Multiple linguistic terms

H_S={very_low, low, medium}

between very_low and medium
or
lower than medium

- Facilitate the expression of linguistic preferences
- Propose the use of Context-free grammars
  - Formalization of linguistic expressions
  - Introduced by N. Chomsky
  - Used in computation sciences

CHALLENGES: Enrich vocabulary

- HFLTS → Multiple linguistic terms

H_S={very_low, low, medium}

between very_low and medium
or
lower than medium

- To facilitate the expression of linguistic preferences
- Propose the use of Context-free grammars

A context-free grammar G is a 4-tuple (V_N,V_T,I,P), where
V_N is a set of nonterminal symbols,
V_T is the set of terminal symbols,
I is the starting symbol and
P are the production rules defined in an extended Backus-Naur form.

CHALLENGES: Enrich vocabulary

- Basic context-free grammar

Let G_H be a cf grammar and S a linguistic term set. $G_H = (V_N, V_T, I, P)$,

$$V_N = \{\langle primary\ term\rangle, \langle composite\ term\rangle, \langle unary\ relation\rangle, \langle binary\ relation\rangle, \langle conjunction\rangle\}$$

$$V_T = \{lower\ than, greater\ than, at\ least, at\ most, between, and, s_0, s_1, \ldots, s_g\}$$

$$I \in V_N$$

$$P = \{I ::= \langle primary\ term\rangle | \langle composite\ term\rangle$$

$$\langle composite\ term\rangle ::= \langle unary\ relation\rangle \langle primary\ term\rangle | \langle binary\ relation\rangle \langle primary\ term\rangle \langle conjunction\rangle \langle primary\ term\rangle$$

$$\langle primary\ term\rangle ::= s_0 | s_1 | \ldots | s_g$$

$$\langle unary\ relation\rangle ::= lower\ than | greater\ than | at\ least | at\ most$$

$$\langle binary\ relation\rangle ::= between$$

$$\langle conjunction\rangle ::= and\}$$

CHALLENGES: Enrich vocabulary

- Context-free grammar provides flexibility
  - Its definition → Specific problem

Example:

S={s_0:nothing,s_1:very low,s_2:low,s_3:medium,s_4: high,s_5:very high,s_6:perfect}

- Comparative linguistic expressions
  - ll_1: high
  - ll_2: lower than low
  - ll_3: greater than high
  - ll_4: at least high
  - ll_5: at most low
  - ll_6: between medium and very high

CHALLENGES: Enrich vocabulary

- Manage comparative linguistic expressions
  - HFLTS
- Transformation function, $E_{G_H} : ll \longrightarrow H_S$
  - Linguistic expressions transformed into HFLTS to operate

Richer Grammars ?

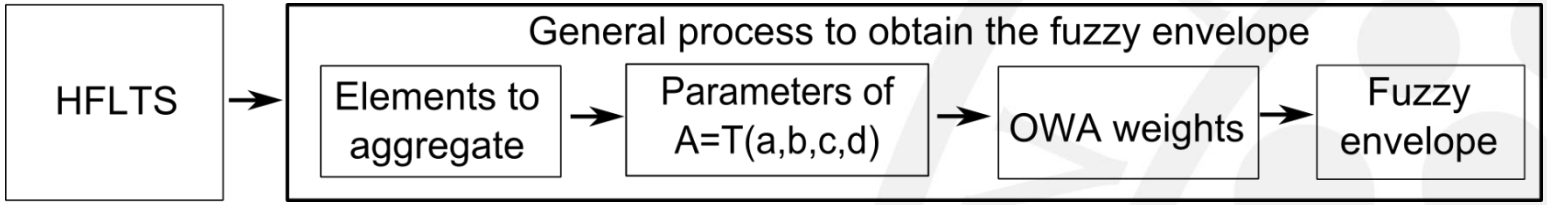
- $E_{G_H}(s_i) = \{s_i | s_i \in S\}$
- $E_{G_H}(\text{at most } s_i) = \{s_j | s_j \in S \text{ and } s_j \leq s_i\}$
- $E_{G_H}(\text{lower than } s_i) = \{s_j | s_j \in S \text{ and } s_j < s_i\}$
- $E_{G_H}(\text{at least } s_i) = \{s_j | s_j \in S \text{ and } s_j \geq s_i\}$
- $E_{G_H}(\text{greater than } s_i) = \{s_j | s_j \in S \text{ and } s_j > s_i\}$
- $E_{G_H}(\text{between } s_i \text{ and } s_j) = \{s_k | s_k \in S \text{ and } s_i \leq s_k \leq s_j\}$

CHALLENGES: Enrich vocabulary

- Fuzzy representation for HFLTS
- Fuzzy linguistic approach
  - Linguistic terms
    - Syntax and fuzzy semantics
  - Semantics of comparative linguistic expressions
    - Fuzzy membership functions
    - Uncertainty and vagueness
- Fuzzy envelope
  - Trapezoidal fuzzy membership function
  - Multiple linguistic terms of HFLTS

CHALLENGES: Enrich vocabulary

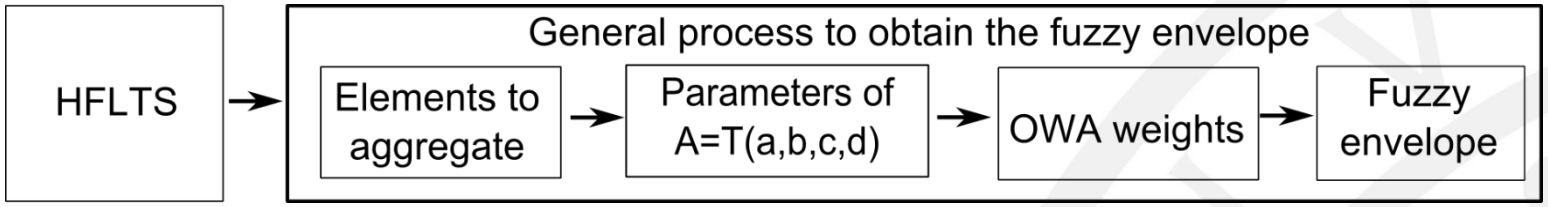
- Fuzzy representation for HFLTS
  - Trapezoidal fuzzy membership function
    - Aggregation of fuzzy membership functions
    - According to their relevance



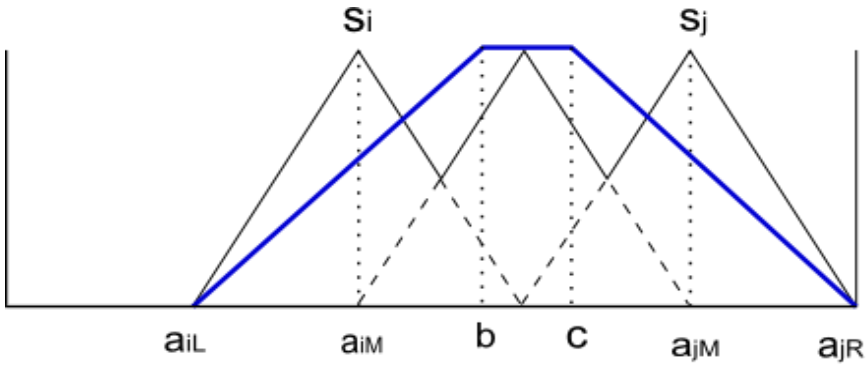
R.M. Rodríguez, H. Liu, A fuzzy envelope for hesitant fuzzy linguistic term set and its application to multicriteria decision making. Information Sciences. 2013.http://dx.doi.org/10.1016/j.ins.2013.07.027

CHALLENGES: Enrich vocabulary

- Fuzzy representation for HFLTS
  - Trapezoidal fuzzy membership function



Obtain the Fuzzy Envelop



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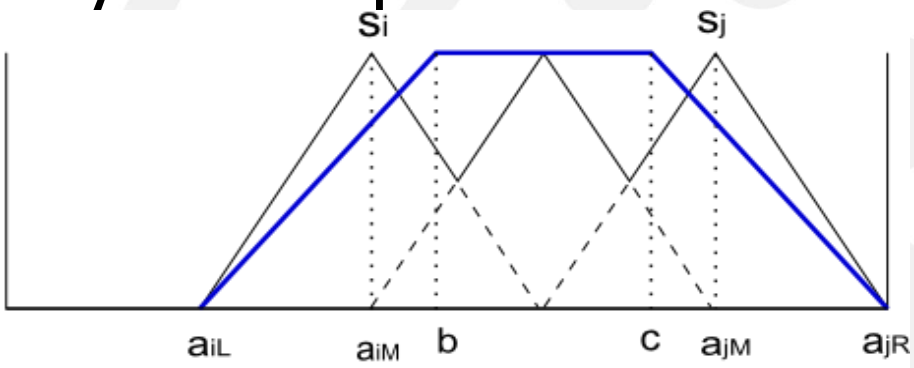

(ii)

CONCLUSIONS & FUTURE RESEARCH

- CWW useful and necessary tool in DM
  - Many models to successful solve DM under uncertainty
  - Bring closer human beings capabilities and machine processes
- Still challenges
  - Better semantics representation
  - Richer vocabularies
  - ***Decision Analysis***

Thank you for your attention